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Operational Definition of Coherence

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Abstract

An essentially important concept in quantum mechanics, *coherence* is discussed from an operational viewpoint of the difinition. In a typical interferometer, i.e., Si interferometer for neutron beams, superposition occurs between separate beams only for the equi-energy condition. However, the so-called quantum beat phenomenon insists that superposition is made on beams with different energies and the phenomenon results in a beating oscillation with a difference frequency between both deexcitation processes. Here we invoke a new quest: Are the states with different energies really incoherent?

1. Introduction

A single wave representing a single particle like electron or neutron can be decomposed into at least a pair of beams, since it is represented by a wavefunction, i.e., probability amplitude. So long as the decomposed waves experience the same environment and nothing else, then both beams continue to have the same physical quantities, esp., the same energy. In such cases there is no problem to let them interfere again.

However, when one beam suffers from a different environment, causing its energy different from that of the other, a serious problem occurs. Do they still keep coherence and interfere to each other with different energies?

Here we discuss exclusively the coherence of the first order. Coherence of higher orders, such as that observed in the well-known Hanbury Brown-Twiss type experiment must be of another interest and importance. However, the very basic notion of coherence seems to manifest itself in the lowest order phenomenon. In other words, we are concerned only with the coherence of a particle to itself in the present paper.

2. Interferometry Experiment

In quantum mechanics coherence is observable in interferometry experiment. Electron beams are emitted from an specifically prepared "coherent" electron source (usually a field emission electron gun) and are decomposed using an Aharonov-Bohm type solenoid. A tiny current through the solenoid varies the phase difference between both beams, each of which passses either the left-side or right-side of the solenoid.

For the neutron interferometer carved out of a bulky single crystalline silicon (Fig. 1), an incident beam is decomposed into the transmitted and reflected directions caused by the Bragg diffraction condition.

In both decomposed particle states, say, ψ_1 adn ψ_2, as is ordinarily described in any textbooks, a superposed state is given by

$$\psi = \psi_1 + \psi_2, \tag{1}$$

so long as both states are coherent. The plus sign between the states is only a mathematical symbol and loses its physical validity when the two states are not coherent in the physically well-defined meaning. That is, *incoherent* states may not be superposed and an interference fringe is no longer expected, so that Eq.(1) is invalidated.

In recent researches on mesoscopic systems in condensed-matter physics, it has become gradually recognized that Bloch electrons remain to be coherent unless they suffer from inelastic scattering. For any states to lose coherence, elastic scattering plays no important role. This fact insures that Bloch electrons can maintain coherence although there exist many and many kinds of elastic scattering events in actual condensed matter.

From this recognition, we can infer that coherence occurs only for equi-energy states. This fact holds surely on neutron interferometry, too.

As shown in Fig.2, superposition of two neutron beams occurs when the Bragg condition is necessarily satisfied for both beams. This result was obtained by simulation (Murayama (1989), (1990), (1991)). If the incident two beams do not satisfy the same Bragg condition of diffraction, superposition does not occur on the same superposer. That is, when one of the beams has a different energy from the other, and, hence, the energies

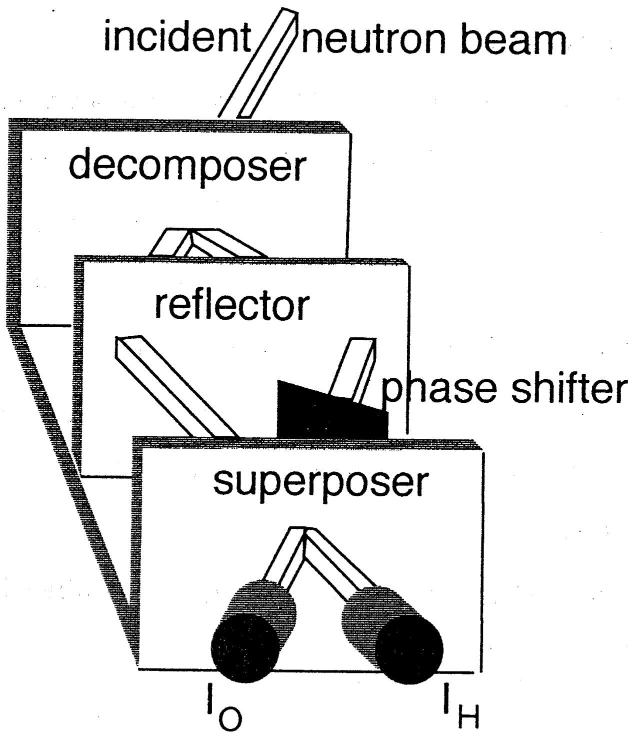

Fig.1. Schematic illustration of neutron interferometer made of single crystalline silicon. There are three platelets, atomically parallelly configured, playing as decomposer, reflector, and superposer. A phase shifter performs by being rotated around an axis, that causes an effective path length to be different from that in the phase-shifter-free region. Whether superposition occurred or not is known by calculating both intensitied I_O and I_H as a function of phase difference generated by the phase shifter. The sum of both intensities is kept constant, whereas each intensity varies sinusoidally with the phase difference.

of the two are different, both states look to behave incoherently.

In an electron interferometer it is ordinary to utilize a fluorescence plate to detect whether an electron really reached there. In such case the detector plays at the same time the role of superposer as well. In high energy experiments monochromaticity is ordinarily given with a ratio $\Delta E/E$, so that ΔE amounts to a considerable magnitude for a large E value. To excite one fluorescence atom the necessary energy should be definite within an energy width around a rather small value, typically $\sim$2.5 eV. However, an incident electron beam usually has a much higher energy than that necessary to excite one fluorescent atom. In this meaning it is difficult to claim that the condition of equi-

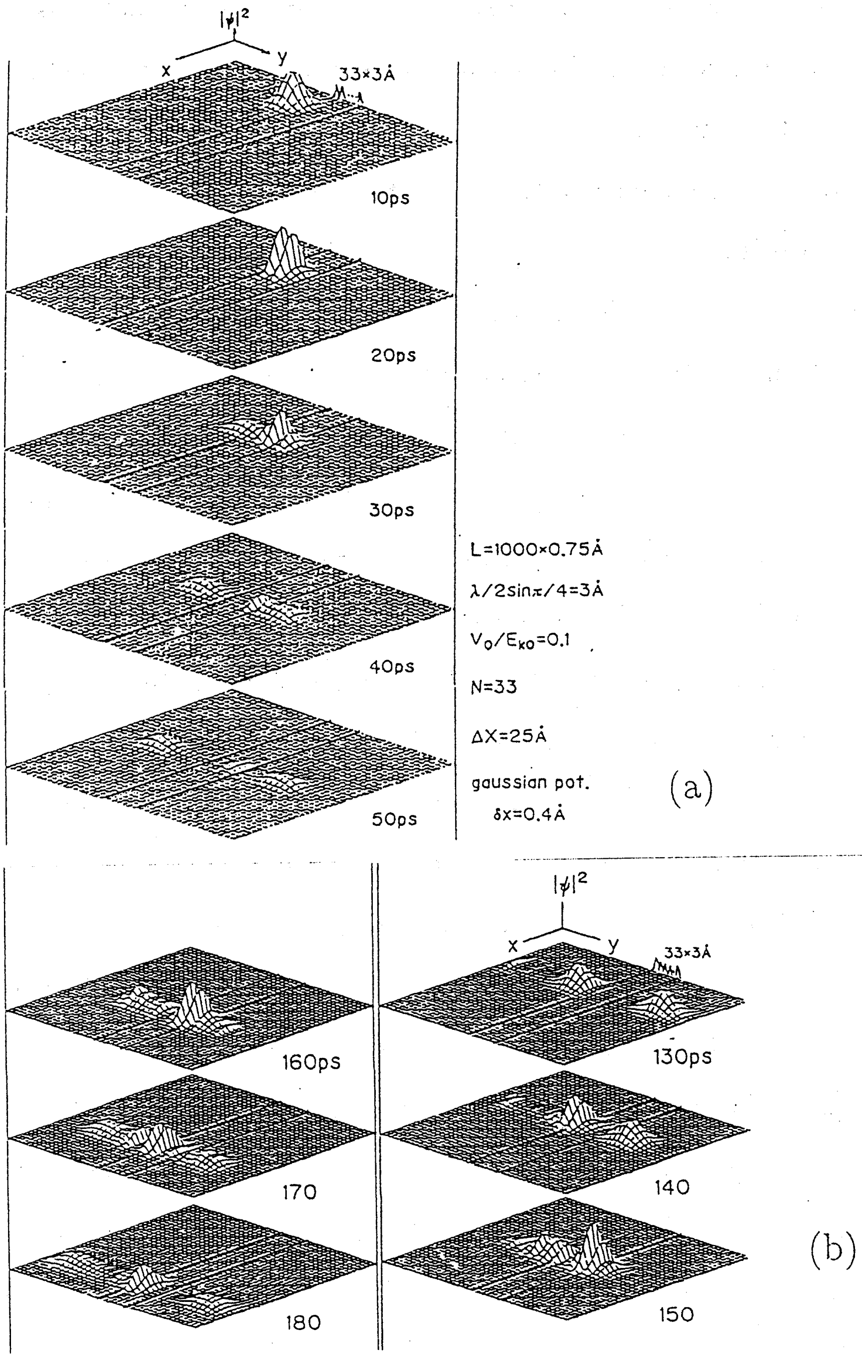

Fig.2. Simulated performance results of a silicon neutron interferometer. The regions enclosed with the parallel lines are assumed to have periodic potentials to exert scattering on the incident neutron. The first platelet of silicon plays the role of decomposer, the second one plays simply that of reflector, and the last one that of superposer. All platelets must satisfy the same Bragg condition to realize required performance. To have the beams satisfy the same condition, both beams must have the same energy.

energy to cause interference is strictly satisfied in electron interferometry as well.

So long as a silicon interferometer is concerned, the concept of coherence seems to be derived only for the same energy for the pair of beams. For electron beams it seems slightly difficult to decisively conclude the same statement.

3. Converse Discussion

When we discuss coherence, it is obvious that both beams are coherent when they have the same energy. Then, how may we discuss the case with different energies? In a silicon interferometer, we already discussed that one of the incident beams pass through the platelet without interacting considerably, unless it satisfies the Bragg condition, so that both beams with different energies do not interfere. However, the equi-energy condition seems to be only a sufficient condition, not a necessary one.

To testify this fact, let us remind that there exists one example: a quantum beat phenomenon (for essay, refer to: Silverman (1993)). Figure 3 shows a schematic interpretation of the phenomenon. In this process, an interference of the probability amplitudes, say, wave functions, occurs between a pair of possible excitation-deexcitation processes. In contrast to that there were two possible *spacial* paths from the decomposer to the superposer in the silicon interferometer, here are two possible *temporal* paths from the initial to the final state in this case. In order to assure coherence between the two paths, coherent excitation to the intermediate states is essentially important. Experimentally, this is exerted by using an ultra short laser pulse. That is, a short pulse of duration ΔT has an energy uncertainty of the order of $\Delta E \sim \hbar/\Delta T$ and if ΔE covers the two or more intermediate energy states, an electron may be excited into the superposed state of those intermediate states.

This phenomenon described so far is certainly related to the coherence between the states with different energies. Coherent excitation must be possible for the more different energy states, is the shorter pulse utilized. This consideration seems to let us assert that the states with different energies may behave coherently as well.

As a matter of course this kind of coherence should be observable only when appropriate experiments are devised. In other words, coherence depends on the experimental

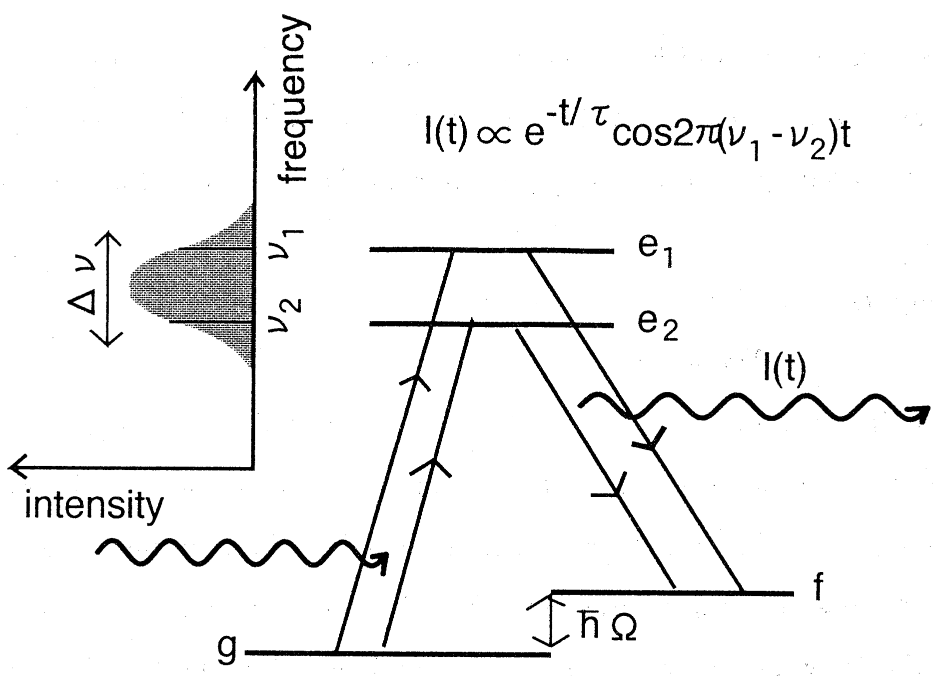

Fig.3 A schematic illustration of quantum beat phenomenon. If intermediate states e_1 and e_2 are coherently excited, both paths: $g \to e_1 \to f$ and $g \to e_2 \to f$ must be considered to superpose coherently in the probability amplitude, that causes an oscillatory emission intensity between the intermediate and final state. The period of the oscillation is $1/\nu = h/(E_{e_1 f} - E_{e_2 f}) = 1/(\nu_1 - \nu_2)$. $1/\tau$ is the decay rate.

setup to observe the coherence in.

Recently a remarkably oscillating quantum beat observation was reported by Höfer, et al. (1997), where two-photon PES (photo-emission spectroscopy) was measured. The first excitation was made with a photon from a pumping laser of an energy insufficient to cause photo-emission, but with a pulse ultra short enough to cover several hydrogen-like levels constructed by an image potential at the Cu(100) surface. After a delay time τ a second probing laser pulse was irradiated. In such experiment the delay time plays the same role as the phase shift in the *welcher-weg* interferometry of neutron and electron. In their experiment, the final PE state must be a single state although the intermediate states belonged to different energies. That is, it is essentially important that the PE electron is a single one, not a pair of them, although it experienced a superposition of multiple states with different energies.

Without coherence, the PES would have decayed simply exponentially as a function of τ. In this case, the exponential decay curve was modulated oscillatorily, and they observed a few oscillation periods corresponding to the difference of $n = 4$ and 5 as well as $n = 5$ and 6.

4. Superposition of Excited States

4.1 1-PE Electron+2-Photon Process

The experiment cited above is for a PE electron emitted after two-photon excitation. There, intermediate states are excited coherently and two possible paths were for $g - e_1 - PE$ and $g - e_2 - PE$. Photo-electrons generated by the possible two processes were collected and measured, and interference "fringe" manifested itself in the spectrum as a function of the delay time between pumping and probing pulses. In this case, superposition occurred over e_1- and e_2-state, not over the final PE-state, and was induced to the couple of states by the same electron-photon interaction brought by a probing laser pulse.

In order that coherent quantum beat may occur, it is necessary for the electron excited in e_1 and e_2 not to experience any considerable dephasing relaxation. In other words, dephasing or decoherence, sometimes equivalent to inelastic scattering, is the same process that the electron loses its memory of phase. The quantum beat in this experiment is expected to appear only within the duration of the delay τ less than the smallest dephasing time.

Anyway, this is a real example of coherence between the states with different energies, so long as they can be excited coherently.

4.2 1-Electron+2-Photon Process

In the explanation of Fig. 3, we stressed that the excitation into a superposed state must be made with an ultra short pulse. It is according to the very coherence that the quantum beat occurs caused by the superposition of states over e_1 and e_2. Awidely energy-dispersed single photon can excite an electron into superposable states with different energies. This excitation is essentially a single-photon process. If superposition occurs in the f-state, then no beat would occur. The superposer in this experimentis the electron-photon interaction itself.

5. Summary

Coherence is a necessary condition for the states to superpose. However, as we already discussed, an actual physical process is needed to cause superposirtion. Thus, coherence is a conceptual notion, and superposition or interference is a physical process.

Let us consider an alternative situation. In nonlinear optics, the so-called up-conversion process is well known. For example, an electron can be excited by a photon $h\nu_1$ into an excited state e_1. Likewise, the electron can be excited by "coherent" two-photon absorption process, namely, from g to e_1 and subsequently to e_2. The second excitation is caused by absorbing the second photon $h\nu_2$. Eventually from the excited e_2 to f, an up-converted photon appears with an energy equal to $h(\nu_1 + \nu_2)$. Similarwise, from e_1 to f, another photon comes out. Are these two processes not coherent? Cannot state e_1 and e_2 be superposed? Why not?

Each process is a coherent process. However, there is no reason for the both to be coherent, since both processes are separate. State e_1 is for one electron, and e_2 for the other. Accordingly, these two processes are with two electrons. Even if there should exist a second-order coherence, the first-order one does not. The basic coherence is only produced by a single-particle process.

In summary, coherence is exclusively observable through some superposition process, which is an actual physical process and an actual physical device is needed to perform superposition.

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